Superbubbles, Galactic Dynamos and the Spike Instability

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1 Abstract

We draw attention to a problem with the alpha-Omega dynamo when it is applied to the origin of the galactic magnetic field under the assumption of perfect flux freezing. The standard theory involves the expulsion of undesirable flux and, because of flux freezing, the mass anchored on this flux also must be expelled. The strong galactic gravitational field makes this impossible on energetic grounds. It is shown that if only short pieces of the undesirable field lines are expelled, then mass can flow down along these field lines without requiring much energy. This expulsion of only short lines of force can be accomplished by a spike instability associated with gigantic astrophysical superbubbles. The physics of this instability is discussed and the results enable an estimate to be made of the number of spikes in the galaxy. It appears that there are probably enough spikes to cut all the undesirable lines into pieces as short as a couple of kiloparsecs during a dynamo time of a billion years. These cut pieces then may be randomly rotated in a dynamo time by alpha-Omega diffusion and there is enough rotation to get rid of the undesirable flux without expelling the fields themselves. The spike process seems strong enough to allows the alpha-Omega dynamo to create the galactic field without any trouble from the boundary condition problem.

2 Introduction

Our galaxy is believed to have finite magnetic flux in the form of a toroidal field, because this field does not reverse across the galactic midplane. The origin of this field is paradoxical. This is because, on the galactic scale, flux freezing is almost infinitely strong so that flux through any moving plasma

region cannot change. In fact, the change is only finite over a Hubble time if the scale of variation is smaller than an astronomical unit. Thus, the obvious question arises: How can one start with a weak field of cosmological origin and increase it to its present value?

The answer is: that the flux in the galactic disc itself need not be constant, but only the flux in the larger region consisting of the disc plus the galactic halo. The standard alpha-Omega theory (Steenbeck et. al [1]) supposes one starts with a very weak field whose origin is cosmological, and is amplified by compression during galactic formation. (Such an initial field might have a strength of $B_0 = 10^{-12}$ gauss, be toroidal in the positive direction, and uniformly fill the disc with flux Φ.) The standard alpha-Omega dynamo folds this field back and forth by the alpha effect. The result is a flux of 2Φ in the positive direction about the midplane and a negative flux of $-\Phi$ near the edges of the disc. Then the negative flux is supposed to be turbulently diffused out of the disc and into the halo leaving the disc with double its original flux. This doubling takes place during a 'dynamo time' of less than a billion years. It repeats over and over perhaps twenty times in the life of the galactic disc amplifying the field strength (and flux in the disc) by over a million times. Such a process appears to provide a reasonable origin of our present galactic field, and also satisfies the magnetic flux freezing condition, (Parker [2], Ruzmaikin, et. al. [3]).

However, when one looks more closely at the diffusion of flux from the disc to the halo, a problem arises because the galaxy has a very strong gravitational field. In the motion of a flux tube into the halo, the mass anchored onto it by flux freezing must also be lifted into the halo.

During this motion an energy, equivalent to an escape velocity of four hundred kilometers a second, must be supplied to the mass. This energy is extremely large compared to energies in interstellar turbulence, which have typical velocities of ten kilometers a second. Where can such energies come from?

One suggestion is supernovae. But velocities of supernovae remnants are reduced to a few tens of kilometers a second by snow plowing all the surrounding interstellar mass. They have only a small chance of breaking out of the galactic disc.

An alternative suggestion is the recently recognized phenomena of super-bubbles, which are driven by multiple supernova. Some of these actually break out of the galactic disc. However, by the time they leave the disc, the mass that they have snow plowed has slowed them down to a velocity of

fifteen to twenty kilometers a second. So this can not be a direct source of the large energies needed for diffusion of the tubes into the halo.

One is forced to consider a different concept. This concept is based on the realization that the flux freezing process applies only to motions perpendicular to the line of force. It in no way constrains the parallel motion along the field. Thus, if a short length ℓ_S of a line of force is lifted out of the disc and into the halo, the mass anchored on it can be greatly reduced from its initial value by sliding down along a length ℓ_V to the rest of the line ℓ_R remaining in the disc. Because of the small mass on the ℓ_S little energy is required. Thus, on energy grounds alone, such a process is possible. But removing a short piece of a negative line into the halo does not itself provide a reduction in the negative toroidal flux in the disc. A further idea is needed (Kulsrud [4]).

Consider what happens to the rest of the negative field line ℓ_R on either side of ℓ_S? There is a gap in it caused by the removal of ℓ_S. Of course, this gap is kept closed by two vertical pieces of the field ℓ_V, connected to ℓ_s. But the field strength of these pieces, ℓ_V, becomes very weak by horizontal expansion. The region into which these $\ell_V s$ expand is much larger than the thickness of the disc by a factor of fifty, the aspect ratio of the disc, so that the field strength of these ℓ_V become far too weak to affect the rest of the line, ℓ_R. This part of the line acts as though its two pieces are effectively decoupled or cut. Therefore, the ends of ℓ_R are free to move and it appears that this part of the field line has free ends.

Suppose a given line of force is 'cut' into a number of pieces by removal of a number of short ℓ_S pieces. Then, when these ℓ_{Rs} are acted on by β, the turbulent diffusion of the alpha-Omega dynamo, they will be rotated randomly in direction and no longer preserve their negative toroidal flux. The rotations transfer their negative flux to the ℓ_S pieces in the halo. Due to the weakness of the ℓ_V pieces this transfer through the weak ℓ_V pieces of flux will have no dynamic effect on the disc pieces ℓ_R. But this transfer effectively removes negative flux into the halo as required by the alpha-Omega dynamo. Only a small amount of energy is required for these processes. See figure 1.

It remains to discover a mechanism to take the small ℓ_S pieces and propel

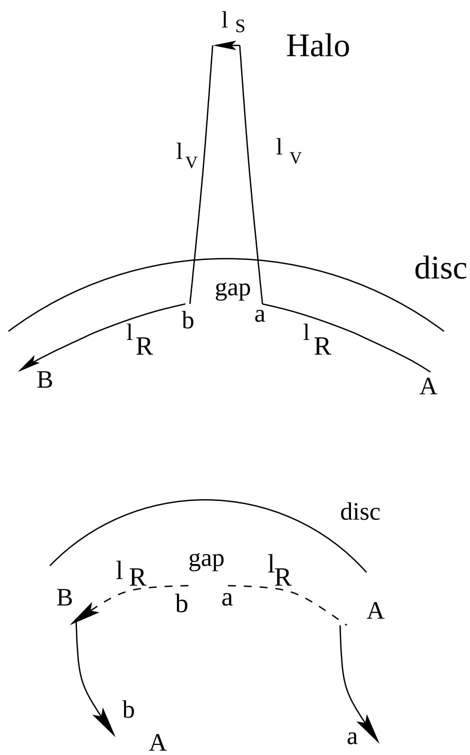

Figure 1: (a) A short piece of a field line ℓ_S is lifted into the halo. It is connected to the pieces ℓ_R in the disc by weak pieces ℓ_V. (b) There is a gap in ℓ_R that allows the ends of the field lines to rotate freely under diffusion so the points a and b rotate to new positions.

them far into the halo. In paper I (Kulsrud [5]) it is proposed that superbubbles present a way to do this.

Each superbubble arises from the formation of a normal galactic star cluster. In the star cluster the stars are formed in a short time and the more massive O and B stars in the cluster continually evolve and explode into supernovae. The many supernova explosions overlap and their energy is transformed into a high pressure low density plasma. Its pressure forces all the surrounding interstellar matter into a gigantic rapidly expanding shell which comprises the superbubble phenomena. The shell gradually slows down as all the surrounding mass accumulates onto it. The shells of the more luminous superbubbles manage to escape out of the disc. Once they leave the disc, their expansion accelerates because the pressure of the core remains large (the supernova are still exploding), and there is no more mass to snowplow.

This situation where low density plasma is accelerating the high density shell is unstable to the Raleigh-Taylor instability, (MacLow and McCray [6]) and the shell breaks up into many fragments. The fragments may still be connected and the pressure still contained by the fragmented shells. This leads to a secondary instability the 'spike instability'.

This instability is described in paper I. The spike is a narrow elongated eruption on a fragment. The matter at the top of the spike is accelerated against gravity, and because of this gravity the mass falls down along the sides of the spike. (See figure 2). As the mass at the top slides down the top becomes lighter and, as long as the pressure is confined, it accelerates faster. This increases the steepness of the sides of the spike and the down flow increases, making the top still lighter and accelerate faster. This process I call the 'spike instability'. As the spike rises towards the halo it increases its speed until finally the velocity exceeds the escape velocity and the top mass reaches the halo ballistically.

The shell mass contains the interstellar field and as the spike rises from the fragment, it carries the ℓ_S pieces of the same lines of force with it. Most of the lines slide down with the mass but some of those at the very top of the spike reach the halo as desired for the model of the dynamo.

For each spike the ℓ_R parts of these lines of force are cut. When enough

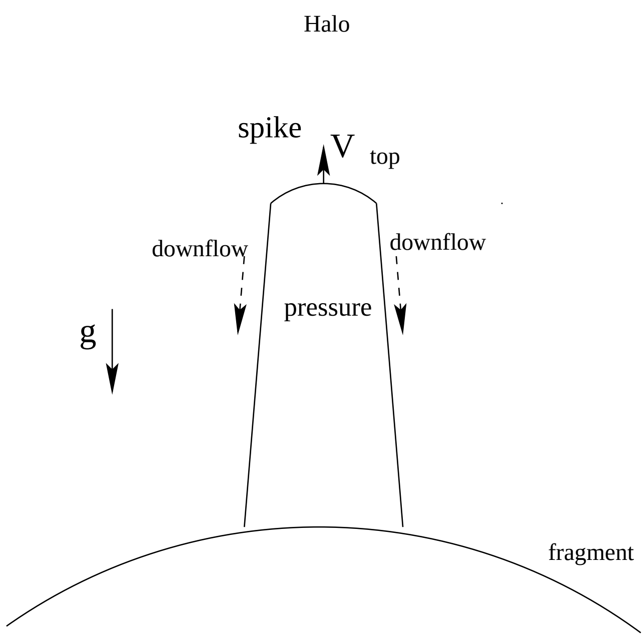

Figure 2: As the spike grows, gravity g leads to a downflow of the mass from the top of the spike to the fragment, making the top lighter. The superbubble pressure then lifts the top faster

lines are cut into short enough pieces the negative flux can be diffused away by random turbulent rotation of these ℓ_R pieces. If this occurs during a short enough time compared to dynamo times (say a billion years), then the alpha-Omega dynamo should work and the galactic field amplified from a weak field to its present value.

3 The Spike Instability

In this section we develop the theory of the spike instability in order to determine what fraction of the magnetic lines of force initially in the spike are expelled to the halo.

Then we estimate the number of spikes and the average length of the cut lines, and finally, we calculate the amount of angular randomization of the ℓ_R lines that occurs during a dynamo time. It from this calculation it appears that the process of flux removal works during a dynamo time of a billion years. In the final section, we summarize and draw the conclusions.

As a model for the spike we approximate the surface of the superbubble fragments as infinitely thin plane sheets with surface mass σ_0 supported against gravity by a constant underlying pressure. Its surface density and actual thickness D are derived in the paper I. The gravitational acceleration g is the sum of the true gravity due to the stars in the disc and the effective gravity from the acceleration of the superbubble shell. However, in our examples the astrophysical gravity is the more important.

We work in cylindrical coordinates with $\hat{\mathbf{z}}$ vertical to the sheet and $\hat{\mathbf{r}}$ and $\hat{\theta}$ in the initial plane of the sheet. We write the equations of the perturbed motion in a Lagrangian form. We assume cylindrical symmetry and assume all quantities depend only on time t and the initial radius of the perturbation r_0. Let $\xi_r(t, r_0)$ and $\xi_z(t, r_0)$ be the components of the displacement.

Let the coordinates of the four corners of an undisplaced small square $\mathbf{A_0}$ at time $t = 0$ be

$$\begin{aligned} P1 &= [r_0, 0, 0] \\ P2 &= [r_0 + \delta, 0, 0], \\ P3 &= [r_0, \epsilon, 0] \\ P4 &= [r_0 + \delta, \epsilon, 0] \end{aligned} \tag{1}$$

Then after the displacement, the coordinates of the four corners of the

displaced square **A** are

$$\begin{aligned}
P1 &= [r_0 + \xi_r(r_0), 0, \xi_z(r_0)] : \\
P2 &= [r_0 + \delta + \xi_r(r_0) + \delta\frac{\partial\xi_r(r_0)}{\partial r_0}, 0, \xi_z(r_0) + \delta\frac{\partial\xi_z(r_0)}{\partial r_0}], \\
P3 &= [r_0 + \xi_r(r_0), \epsilon, \xi_z(r_0)] \\
P4 &= [r_0 + \delta + \xi_r(r_0) + \delta\frac{\partial\xi_r(r_0)}{\partial r_0}, \epsilon, \xi_z(r_0) + \delta\frac{\partial\xi_z(r_0)}{\partial r_0}],
\end{aligned} \tag{2}$$

For notational convenience, let us abbreviate the components of the displacement as

$$\begin{aligned}
\xi_r(r_0) &= R \\
\xi_z(r_0) &= Z
\end{aligned} \tag{3}$$

After the displacement, the components of the vector **A** are

$$\begin{aligned}
A_r &= -\epsilon\delta r Z' = -\epsilon\delta(r_0 + R)Z' \\
A_z &= \epsilon\delta r(1 + R') = \epsilon\delta(r_0 + R)(1 + R')
\end{aligned} \tag{4}$$

where prime denotes a derivative with respect to r_0. (Where convenient we drop the arguments t, and r_0). The vector **A** is perpendicular to the displaced square and its magnitude is equal to its area.

In the equation of motion of the surface of the spike the mass of the square is $\sigma_0|\mathbf{A_0}|$, the pressure force on it is $p\mathbf{A}$, and the gravitational force is $-g\sigma_0|\mathbf{A_0}|\hat{\mathbf{z}}$. From the initial equilibrium we have $g\sigma_0 = p$

Thus, the equations of motion of the displacement are

$$\begin{aligned}
\frac{\partial^2 R}{\partial t^2} &= -g(1 + \frac{R}{r_0})Z' \\
\frac{\partial^2 Z}{\partial t^2} &= g(1 + \frac{R}{r_0})(1 + R') - g
\end{aligned} \tag{5}$$

where the prime denotes the derivative with respect to r_0.

For the linear solution of these equations assume that R and Z are proportional to $\exp\gamma t$. Keeping the linear terms we have

$$\begin{aligned}
\gamma^2 R &= -gZ' \\
\gamma^2 Z &= = g(\frac{R}{r_0} + R')
\end{aligned} \tag{6}$$

Defining

$$k = \frac{\gamma^2}{kg} \tag{7}$$

we find

$$\begin{aligned} Z'' + \frac{Z'}{r_0} + k^2 Z &= 0 \\ R'' + \frac{R'}{r_0} + k^2 R - \frac{R}{r_0^2} = 0 \end{aligned} \tag{8}$$

so the linear solutions regular at $r_0 = 0$ are

$$\begin{aligned} Z &= a e^{\gamma t} J_0(kr_0) \\ R &= a e^{\gamma t} J_1(kr_0) \end{aligned} \tag{9}$$

The growth rate is $\gamma = \sqrt{kg}$. The shortest wave length consistent with an equilibrium sheet mass is given by $kD = 1$ where D is the actual thickness of the layer which we approximated above as a sheet. An estimate of this thickness, which is given in paper I, is about a parsec. The value of g in the neighborhood of the disc is 10^{-8} cm/ sec^2. These values give the linear time of growth, $t_G = 1/\gamma = 7.4 \times 10^5$ years, a time short compared to the life time of the superbubble (usually 20-50 million years).

To get the long time behavior of the spike, we approximate Equation (5) by its nonlinear terms.

$$\begin{aligned} \frac{\partial^2 R}{\partial t^2} &= -\frac{g}{r_0} R Z' \\ \frac{\partial^2 Z}{\partial t^2} &= \frac{g}{r_0} R R' \end{aligned} \tag{10}$$

These equations have a solution with R and Z proportional to $1/(t_0 - t)^2$. Setting

$$\begin{aligned} R &= \frac{\eta}{(t_0 - t)^2} \\ Z &= \frac{\nu}{(t_0 - t)^2} \end{aligned} \tag{11}$$

We have from the first equation,

$$6\eta = -\frac{g}{r_0} \eta \nu' \tag{12}$$

Canceling η from this equation, and solving for ν gives

$$\nu = C - 3\frac{r_0^2}{g} \tag{13}$$

The equation for η is

$$6\nu = \frac{g}{r_0}\eta\eta' \tag{14}$$

which on substituting the solution for ν gives

$$(\eta^2)' = \frac{12C}{g}r_0 - \frac{36}{g^2}r_0^3 \tag{15}$$

and

$$\eta^2 = \frac{6C}{g}r_0^2 - \frac{9}{g^2}r_0^4 \approx \frac{6C}{g}r_0^2 \tag{16}$$

The constants C and t_0 are determined by matching both terms of the solution for Z to its linear solution at $t = 0$.

$$\frac{(C - 3r_0^2/g)}{t_0^2} = a(1 - k^2 r_0^2/4) \tag{17}$$

which gives

$$\begin{aligned} \frac{3}{gt_0^2} &= \frac{k^2 a}{4} \\ \frac{C}{t_0^2} &= a \end{aligned} \tag{18}$$

Solving these two equations for t_0 and C we have

$$\begin{aligned} t_0 &= \sqrt{\frac{12}{k^2 g a}} \\ C &= a t_0^2 = \frac{12}{k^2 g} \end{aligned} \tag{19}$$

so

$$Z = \frac{12}{k^2 g}\frac{(1 - k^2 r_0^2/4)}{(t_0 - t)^2} \tag{20}$$

Treating R the same way we get

$$R == \frac{\sqrt{72}}{kg}\frac{r_0}{(t_0 - t)^2} \tag{21}$$

Now let us determine the time, t_e , at which the upward velocity of the peak of the spike reaches the escape velocity from the of the disc, v_e, (Heiles [7]). That is, from $\partial Z/\partial t = v_e$, (Equation (20)),

$$\frac{\partial Z}{\partial t} = \frac{24}{k^2 g(\Delta t)_e^3} = v_e \tag{22}$$

we get

$$(\Delta t)_e = t - t_e = \left(\frac{24}{k^2 g v_e}\right)^{1/3} \tag{23}$$

.

This occurs at the height

$$h_e = 1.44 \left(\frac{v_e^2}{k^2 g}\right)^{1/3} \tag{24}$$

With the above numbers $h_e = 49$ parsecs.

4 The Number of Lines Cut by One Spike

To determine the number of lines expelled we start with the number of lines initially embedded in the spike. This is equal to the number of lines embedded in the superbubble shell in a strip with the width of one spike, $\approx \lambda = 1/k$ Since the shell consists of all the interstellar mass swept up by the superbubble, the strip contains $B\lambda H$ lines, where B is mean field in the disc. At the escape time t_e, the region $r_0 < \lambda$ has horizontally expanded a distance

$$\frac{R(t_e, \lambda)}{r_0} = \frac{\sqrt{72}}{kg}\frac{1}{(\Delta t)_e^2} = \frac{72^{1/2}}{(24)^{2/3}}\left(\frac{kv_e^2}{g}\right)^{1/3} = 37.59\text{pc}. \tag{25}$$

This expansion reduces the field strength (i.e. the density of the lines) at the top of the spike, by a factor of

$$\frac{r_0}{R(t_e, \lambda)} = .0266 \tag{26}$$

However, by the time the spike has reached a height of h_e, the solution of the spike has spread out a distance of 37 parsecs, and the behavior of the spike a distance $r \gg \lambda$ from the cylindrical axis is uncertain. Therefore, we

take for the number of expelled lines, Φ_e, the number of lines at the top of one spike, within $r_0 < \tau\lambda$. (Those that are certainly expelled.)

$$\Phi_e = 2 \times .0266 \times BH\tau\lambda \tag{27}$$

The other lines may fall back down, or possibly be expelled.

Among the these lines there are both positive and negative lines. The proportions depend on how high above the galactic midplane the superbubble started. Also, the sheet of plasma, assumed in our derivation to be infinitely thin, actually has a finite thickness. In the radial outflow flow, the mass in the sheet nearest the superbubble pressure will probably be squeezed out faster than that farther away. Without analyzing this further we simply assume that half the number of lines at the top of the spike are negative and ignore the positive lines. Thus, we take the number of negative lines cut per spike to be

$$\frac{1}{2}\Phi_e = .0266BH\lambda\tau \tag{28}$$

Denote the number of spikes per fragment as f_S, and the number of fragments per superbubble as N_f. The size of the spike is $\lambda \sim D$ where D is the thickness of the superbubble shell.

Consider a superbubble of luminosity $\bar{L} = 1.37 \times 10^{37}$ ergs per second, This is the luminosity of a superbubble just strong enough to break out of the disc, and fragment. For such a superbubble we have, from paper I, that

$$\begin{aligned} N_f &= \frac{78}{T_{300}^2} \\ D &= \frac{H}{222}T_{300} \end{aligned} \tag{29}$$

where the temperature, $T = 300 \times T_{300}$ degrees Kelvin, is the temperature in the superbubble shell. T is generally assumed to be between a hundred and a thousand degrees Kelvin. Then our estimate for the number of cuts of negative lines by a single superbubble Φ_{sb} is

$$\Phi_{sb} = \frac{1}{2}\Phi_e N_f f_S = .0266BDHN_f f_S\tau \tag{30}$$

Substituting the numbers from Equation (29)

$$\Phi_{sb} = .0266\frac{78}{222}\frac{BH^2 f_S\tau}{T_{300}} = 0.00935\frac{BH^2 f_S\tau}{T_{300}} \tag{31}$$

Now what is the number of lines cut by a superbubble with luminosity $L > \bar{L}$? Again from paper I we find that $N_f \sim (\bar{L}/L)^{2/3}$ and $D \sim (L/\bar{L})^{1/3}$ so the number of negative lines cut by a luminosity-L superbubble is

$$\Phi_L = \Phi_{sb} \left(\frac{L}{\bar{L}}\right)^{-1/3} \tag{32}$$

Consider an annular region in the galaxy with mean radius $R_S = 8.5$ kpc (the solar galactic radius) and radial thickness ΔR. The rate of birth of superbubbles with luminosity in dL is (Ferrière [8]).

$$d\sigma = 0.86 \times 10^{-7} \left(\frac{L}{\bar{L}}\right)^{-2.3} \frac{dL}{\bar{L}} \text{ kpc}^{-2} \text{ yr}^{-1} \tag{33}$$

and the rate of birth in the entire annulus is

$$2\pi R_S \Delta R d\sigma \tag{34}$$

so that the rate of line cutting by all the superbubbles with luminosity L in the annulus is

$$\Phi_L d\sigma 2\pi R_s \Delta R \tag{35}$$

Now, the number of magnetic lines of force in the annulus, assuming the lines are toroidal, is $2BH\Delta R$. But only a quarter of these lines are negative, so their number, ψ_B, is

$$\psi_B = .5BH\Delta R \tag{36}$$

The rate of cutting of any given line ρ_C by all the superbubbles with luminosity greater than $\bar{L}$ is the rate of cutting by all these superbubbles (integrated over their luminosity) divided by the number of lines, ψ_B.

$$\begin{aligned}
\rho_C &= \frac{2\pi R_s \Delta R \int_{L>\bar{L}} \Phi_L d\sigma \Phi_L}{.5BH\Delta R} && (37)\\
&= 4\pi \times .00985 \times (.86 \times 10^{-7}) R_S H \int_{L>\bar{L}}^{\infty} \frac{dL}{\bar{L}} \left(\frac{L}{\bar{L}}\right)^{-2.3-1/3} \\
&= 4\pi \times .00935 \times (.86 \times 10^{-7}) \times R_S H \frac{f_S \tau}{T_{300}(1.3 + 1/3)} \\
&= .17 \times 10^{-7} \frac{f_S \tau}{T_{300}(1.3 + 1/3)} \text{ yr}^{-1} \\
&= 10.5 \frac{f_S \tau}{T_{300}} \text{ cuts per billions years}
\end{aligned}$$

Thus, in a billion years, each line of length $2\pi R_S = 53$ kiloparsecs will be cut into $10.5 f_S \tau / T_{300}$ pieces of lengths $\ell = 5.1 T_{300} / f_S \tau$ kiloparsecs.

These lines will be rotated through an angle $\Delta\theta$ by the same turbulent diffusion β as included in the normal alpha-Omega dynamo. The rate of spread in angle is

$$\frac{(\Delta\theta)^2}{2} = \frac{\beta t}{\ell^2} \tag{38}$$

Take $\beta = \beta_{26} \times 10^{26}$ cm^2/ sec. $\beta_{26} \approx 0.5$ (Ferrière [8]). Then

$$\begin{aligned} \Delta\theta &= 0.52 \frac{\sqrt{\beta_{26} t_9}}{\ell_{kpc}} \\ &= 0.11 \sqrt{t_9} \frac{f_S \tau}{T_{300}} \end{aligned} \tag{39}$$

where t_9 is the time in billions of years and ℓ_{kpc} is the length of the cut pieces of lines in kiloparsecs.

This, estimate indicates that the boundary condition on the alpha-Omega dynamo may still be a problem. The uncertainties of the estimate are encapsulated in the parameters $f_S, \tau,$ and T_{300}. If the temperature T in the superbubble shell were as low as 100 degrees, and the parameter τ was as large as 3, then the random rotation θ would be of 0.9. This value for θ is large enough to satisfy the alpha-Omega galactic dynamo with a dynamo time of a billion years. This choice of the parameters is quite reasonable.

It should be noted that, in this model for flux expulsion, the galactic magnetic field has been assumed weak enough that any magnetic tension is too weak to interfere with the spike instability. This limit on the field is reasonable since the main problem in the origin of our galactic magnetic field is how to amplify it when it is extremely weak. When the galactic field reaches its present value, presumably this back reaction of the field will saturate the flux expulsion process and end any further growth of the field.

5 Conclusion

The purpose of this paper is to form a more precise estimate of the flux expulsion by spikes associated with superbubbles, than was given in paper I. The nature and physics of the spikes certainly introduces considerable uncertainty into the conclusions as to whether superbubbles can produce the

required destruction of negative flux needed for the dynamo to act on a time scale of a billion years. In this note we present a conservative estimate of the superbubble process that shows that one *should.* take the spikes seriously in dynamo theory. A multi-dimensional simulation of the spikes is clearly necessary to properly evaluate whether this process is a reasonable way to make the alpha-Omega dynamo viable for our galaxy. However, it should be noted that there are serious difficulties with the dynamo itself, and at the moment there does not seem to be any other reasonable alternatives.

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